Proposal of a Power Efficient N-Level Multipulse PPM-LQAM Technique

Hossam M. H. Shalaby, *Senior Member, IEEE*

***Abstract*—A new power efficient technique, that combines N-level multipulse pulse-position modulation with L-level quadrature-amplitude modulation, denoted by NMPPM-LQAM, is proposed. Its constrained power efficiency is derived and characterized. It is shown that the proposed combination provides a superior technique that is both power and spectral efficient simultaneously. The constrained power efficiencies of variations to this technique are considered and characterized as well. These include combining multilevel pulse-amplitude modulation with LQAM (NPAM-LQAM). The obtained constrained power efficiencies are compared numerically to that of traditional LQAM and MPPM-LQAM techniques. Our results disclose that proposed NMPPM-LQAM is the most power/spectral efficient technique. For example, at spectral efficiency constraints of 2.5 and 4.2 bs^{-1}/Hz/pol, the corresponding constrained power efficiencies of proposed NMPPM-LQAM technique are higher by about 4.87 and 3.62 dB, respectively, than that of traditional LQAM. In addition, the results reveal that NPAM-LQAM scheme has increased power and spectral efficiencies compared to that of traditional NPAM, but its power efficiency is less than that of traditional LQAM.**

***Index Terms*—Constrained power efficiency, multipulse pulse-position modulation, power efficiency, pulse-amplitude modulation, pulse-position modulation, quadrature-amplitude modulation, spectral efficiency.**

I. Introduction

In order to increase the data transmission rate at a given bandwidth, while keeping the bit-error rate reduced, both spectral and power efficiencies are to be optimized simultaneously. Although spectral efficiency optimization has been the main goal since last decade, power efficiency is becoming an important performance measure in recent years. Indeed, an increase in the energy efficiency by 100 times is one of the 5 G requirements, as indicated by the International Telecommunications Union (ITU) report [1], [2].

Multilevel signaling, e.g., quadrature-amplitude modulation (QAM), along with polarization-division multiplexing (PDM) are normally adopted in optical communications systems to increase the spectral efficiencies. Recently, the power efficiencies of multilevel optical signals are increased by adopting hybrid modulation formats, e.g., polarization-switched quadrature phase-shift keying (PS-QPSK) and PDM-QPSK superimposed on pulse-position modulation (PPM) signals [3], [4]. It has been shown that these modulation formats can raise the power efficiencies above that of PDM-QPSK. However, this is gained at a cost of lowered spectral efficiencies. In [5]–[10], it has been demonstrated that both spectral and power efficiencies can be raised above pure QPSK or QAM by combining multipulse pulse-position modulation (MPPM) with multilevel signaling.

In [10], we have characterized the optimum achievable power efficiency under a restriction on the spectral efficiency of MPPM-LQAM technique. Recently, new hybrid techniques have been proposed in literature, namely, two-level multipulse pulse-position modulation-L-ary differential phase shift keying ($2l$.MPPM-LDPSK) [11] and polarization-assisted L-ary differential phase-shift keying-multipulse pulse-position modulation (PA.LDPSK-MPPM) [12].

In this paper, first we propose a new power efficient technique that combines N-level multipulse pulse-position modulation with L-level quadrature-amplitude modulation, denoted by NMPPM-LQAM. This new technique is even more efficient than MPPM-LQAM. We show that it introduces significant improvement in both spectral and power efficiencies when compared to other techniques. Next, we derive an expression of the constrained power efficiency, defined in [10], for the proposed technique and characterize it. In addition, we determine the constrained power efficiency that can be achieved from an extended version of recent $2l$.MPPM-LDPSK format. The extended version combines both $2l$.MPPM and QAM rather than DPSK. We show that this extended version, denoted by $2l$.MPPM-LQAM, is basically a special case of proposed NMPPM-LQAM scheme. Finally, we propose combining multilevel pulse-amplitude modulation (PAM) with QAM, denoted by NPAM-LQAM, and study its efficiencies. Our results reveal that combining NMPPM with LQAM provides a superior technique that is both power and spectral efficient simultaneously. Indeed, the introduction of leveling in a power efficient formats raises its spectral efficiency as will be discussed below.

The rest of this paper is structured as follows. In Section II, we introduce some preludes and formulate the problem statement. Section III is devoted for the derivation of both the spectral and power efficiencies of proposed NMPPM-LQAM format. Problem formulation and characterization is presented in Section IV, where we provide a solution to the optimum

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Hossam M. H. Shalaby is with the Electrical Engineering Department, Alexandria University, Alexandria 21544, Egypt and also with the Department of Electronics and Communications Engineering, Egypt-Japan University of Science and Technology (E-JUST), Alexandria 21934, Egypt (e-mail: shalaby@ieee.org).

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constrained power efficiency of proposed NMPPM-LQAM technique under a constraint on its spectral efficiency. Variations to this technique are considered in this section too, namely $2l$.MPPM-LQAM, NMPPM, and NPAM-LQAM. Numerical results are presented and discussed in Section V, where we provide comparisons between the constrained power efficiencies of proposed techniques against that of traditional MPPM-LQAM and LQAM techniques. Finally, concluding remarks are given in Section VII.

II. Definitions and Problem Statement

In this section we quote definitions of spectral, asymptotic power, and constrained power efficiencies. In addition, we formulate the problem statement.

A. Spectral and Asymptotic Power Efficiencies

The spectral efficiency η and asymptotic power efficiency γ are defined as [3], [13]:

$$\eta = \frac{\log_2 M}{U/2} \qquad \text{bs}^{-1}\text{/Hz/pol}$$
$$\gamma = \frac{d_{\min}^2 \log_2 M}{4\mathcal{E}_s}, \tag{1}$$

respectively, where M is the number of symbols (or constellation vectors), U is the dimension of a symbol, $\mathcal{E}_s$ is the average symbol energy, and $d_{\min}$ is the minimum Euclidean distance between two vectors in constellation space, given by:

$$d_{\min} = \min_{\substack{\boldsymbol{x},\boldsymbol{y}\in\mathcal{S}(M):\\ \boldsymbol{x}\neq\boldsymbol{y}}} \|\boldsymbol{x}-\boldsymbol{y}\|, \tag{2}$$

where $\|\cdot\|$ denotes the Euclidean norm and $\mathcal{S}(M) = \{\boldsymbol{c}_1, \boldsymbol{c}_2, \ldots, \boldsymbol{c}_M\}$ denotes the set of all possible vectors in the constellation space. For any $k \in \{1, 2, \ldots, M\}$, $\boldsymbol{c}_k = (c_{k1}, c_{k2}, \ldots, c_{kU}) \in \mathbb{R}^U$ is a U-dimensional vector, where $\mathbb{R}$ denotes the set of real numbers.

B. Constrained Power Efficiency

The constrained power efficiency $\theta(S_e)$ is defined as the maximum achievable power efficiency of a modulation technique under a lower-bound constraint S_e on its spectral efficiency [10]:

$$\theta(S_e) = \max_{M:\eta\geq S_e} \gamma(M). \tag{3}$$

C. Problem Statement

In this paper, we propose several hybrid modulation techniques and determine their constrained power efficiencies under spectral efficiency constraints S_e. Specifically, we start by introducing N-level MPPM format, then combine it with QAM scheme to get a very efficient NMPPM-LQAM technique. We also consider combining multilevel PAM with QAM as well as $2l$.MPPM-LQAM scheme.

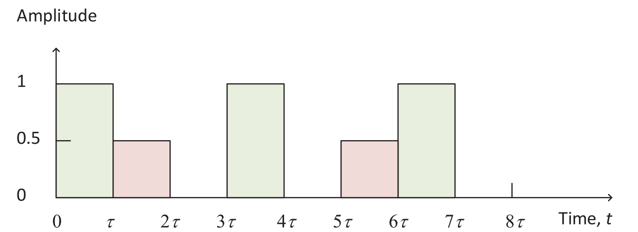


Fig. 1. An example of an NMPPM signal with $K = 8$, $N = 3$, $\boldsymbol{n} = (2, 3)$, and $\alpha_1 = 0.5$.

III. Proposal of NMPPM-LQAM Technique

In this section, we introduce the proposed NMPPM-LQAM format and derive mathematical expressions for its spectral and power efficiencies. We start by a proposal of N-Level MPPM format.

A. Proposal of N-Level MPPM Format

For any integers $K \geq 1$ and $N \geq 2$, we propose the N-level MPPM scheme, denoted by NMPPM, as follows. Given a vector $\boldsymbol{n} = (n_1, n_2, \ldots, n_{N-1}) \in \mathbb{N}^{N-1}$ with $n_1 \geq 1$ and $\sum_{i=1}^{N-1} n_i \leq K$, an NMPPM symbol is represented by a vector $\boldsymbol{d} = (d_1, d_2, \ldots, d_K)$ selected from the set:

$$\mathcal{S}_{N\text{MPPM}}(K, N, \boldsymbol{n}) = \big\{\boldsymbol{d} \in \{0, \alpha_1, \alpha_2, \ldots, \alpha_{N-1}\}^K : f_{\boldsymbol{d}}(\alpha_i) = n_i \,\forall i \in \{1, 2, \ldots, N-1\}\big\}, \tag{4}$$

where $0 < \alpha_1 < \alpha_2 < \cdots < \alpha_{N-1} = 1$ and $f_{\boldsymbol{d}}(x)$ denotes the number of occurrences of x in $\boldsymbol{d}$. Here $\mathbb{N}$ denotes the set of non-negative integers (natural numbers with zero). It is clear that the cardinality of this set equals the multinomial coefficient:

$$\binom{K}{n_1, n_2, \ldots, n_{N-1}, K - \sum_{i=1}^{N-1} n_i} = \frac{K!}{\prod_{i=1}^{N-1} n_i! \left(K - \sum_{i=1}^{N-1} n_i\right)!}. \tag{5}$$

Figure 1 shows an example of an NMPPM signal with $K = 8$, $N = 3$, $n_1 = 2$, $n_2 = 3$, and $\alpha_1 = 0.5$. The pulsewidth is τ and the symbol time frame is $K\tau$. It should be noticed that traditional MPPM format can be considered a special case of NMPPM with $N = 2$.

B. The NMPPM-LQAM Technique

For any integers $K \geq 1$, $N \geq 2$, and $L \geq 2$, we propose the NMPPM-LQAM scheme as follows. Given a vector $\boldsymbol{n} = (n_1, n_2, \ldots, n_{N-1}) \in \mathbb{N}^{N-1}$ with $n_1 \geq 1$ and $\sum_{i=1}^{N-1} n_i \leq K$, an NMPPM-LQAM symbol is represented by a vector $\boldsymbol{c} = (c_1, c_2, \ldots, c_{2K})$ selected from the set $\mathcal{S}_{N\text{MPPM-}L\text{QAM}}(K, N, \boldsymbol{n}, L)$ in (6) at the bottom of this page.

Here, $\mathcal{S}_{\text{QAM}}(L)$ denotes the constellation set of LQAM scheme, where $L = 2^\ell$ and $\ell \in \{1, 2, \ldots\}$ [10]. The cardinality

$$\mathcal{S}_{N\text{MPPM-}L\text{QAM}}(K, N, \boldsymbol{n}, L) = \big\{\boldsymbol{c} \in \mathbb{R}^{2K} : \forall \boldsymbol{d} \in \mathcal{S}_{N\text{MPPM}}(K, N, \boldsymbol{n}) \text{ and } \forall i \in \{1, 2, \ldots, K\}, (c_{2i-1}, c_{2i}) \in d_i \cdot \mathcal{S}_{\text{QAM}}(L)\big\}. \tag{6}$$

of $\mathcal{S}_{N\text{MPPM-}L\text{QAM}}(K, N, \boldsymbol{n}, L)$ is

$$M = L^{\sum_{i=1}^{N-1} n_i} \frac{K!}{\prod_{i=1}^{N-1} n_i! \left(K - \sum_{i=1}^{N-1} n_i\right)!} \tag{7}$$

and the symbol dimension is $U = 2K$.

1) 2l.MPPM-LQAM Format: The $2l$.MPPM-LQAM format, which is an extension of $2l$.MPPM-LDPSK, can be considered as a special case of NMPPM-LQAM with $N = 3$ and $n_2 = K - n_1$. Accordingly, $M = L^K \binom{K}{n_1}$ and $U = 2K$.

C. Spectral and Power Efficiencies of NMPPM-LQAM

The spectral efficiency of proposed NMPPM-LQAM scheme is obtained from (1) as:

$$\eta = \frac{1}{K} \left[\sum_{i=1}^{N-1} n_i \cdot \log_2 L + \log_2 \left(\frac{K!}{\prod_{i=1}^{N-1} n_i! \left(K - \sum_{i=1}^{N-1} n_i\right)!} \right) \right] \text{ bs}^{-1}\text{/Hz/pol.} \tag{8}$$

The average symbol energy $\mathcal{E}_s$ is derived as:

$$\mathcal{E}_s = E\left\{\|\boldsymbol{c}\|^2\right\} = E\left\{\|\boldsymbol{a}\|^2\right\}\|\boldsymbol{d}\|^2 = E\left\{\|\boldsymbol{a}\|^2\right\} \sum_{i=1}^{N-1} \alpha_i^2 n_i, \tag{9}$$

where $E\{\cdot\}$ denotes the expectation value and $\boldsymbol{a} \in \mathcal{S}_{\text{QAM}}(L)$. From [10], we get:

$$\mathcal{E}_s = \frac{2}{3}\left(\kappa_\ell L - 1\right) \sum_{i=1}^{N-1} \alpha_i^2 n_i, \tag{10}$$

where

$$\kappa_\ell = \begin{cases} 1; & \text{if } \ell \text{ is even,} \\ \frac{5}{4}; & \text{if } \ell \text{ is odd and } \ell \neq 1, \\ 2; & \text{if } \ell = 1, \end{cases} \tag{11}$$

with $\ell = \log_2 L$ as mentioned earlier. The minimum Euclidean distance can be derived as:

$$d_{\min} = 2 \cdot \min_{i \in \{1,2,\ldots,N-1\}} \{\alpha_i - \alpha_{i-1}\}, \tag{12}$$

where we set $\alpha_0 = 0$. We select the α_i's so that $d_{\min}$ is maximized. It is easy to check that $d_{\min}$ is maximized at:

$$\alpha_i = i/(N-1), \qquad i \in \{1, 2, \ldots, N-1\}. \tag{13}$$

Accordingly,

$$d_{\min} = 2/(N-1). \tag{14}$$

Using (1), we obtain the power efficiency as:

$$\gamma = \frac{3/2}{\left(\kappa_\ell L - 1\right) \sum_{i=1}^{N-1} i^2 n_i} \left[\sum_{i=1}^{N-1} n_i \cdot \log_2 L + \log_2 \left(\frac{K!}{\prod_{i=1}^{N-1} n_i! \left(K - \sum_{i=1}^{N-1} n_i\right)!} \right) \right]. \tag{15}$$

IV. Problem Formulation and Constrained Power Efficiency Characterization

In this section, we formulate the main problem as stated in Subsection II-C and provide the main result in Theorem 1. Problem extensions follow in this section as well.

A. Constrained Power Efficiency of NMPPM-LQAM Scheme

It is clear from the last section that both spectral and power efficiencies of NMPPM-LQAM scheme are functions of $K, N, \boldsymbol{n}$, and L. Accordingly, we rephrase the main problem in Subsection II-B as:

$$\theta\left(S_e, N\right) = \max_{\substack{(K,\boldsymbol{n},L) \in \mathcal{Y}_N: \\ \eta \geq S_e}} \gamma(K, N, \boldsymbol{n}, L), \tag{16}$$

where

$$\mathcal{Y}_N = \left\{ (K, \boldsymbol{n}, L) \in \mathbb{N}^{N+1} : \sum_{i=1}^{N-1} n_i \leq K, \log_2 L \in \mathbb{N}^+ \right\}. \tag{17}$$

Here, $\mathbb{N}^+$ denotes the set of positive integers (natural numbers without zero).

B. Constrained Power Efficiency Characterization

Theorem 1: In NMPPM-LQAM technique, if S_e denotes the spectral efficiency threshold, then the constrained power efficiency as specified in (16) is characterized by:

$$\theta_{N\text{MPPM-}L\text{QAM}}\left(S_e, N\right) = \max_{\ell \in \{\ell_0, \ell_0+1\}} \frac{3/2}{\kappa_\ell 2^\ell - 1} \cdot \frac{\ell \sum_{i=1}^{N-1} p_i + H\left(\boldsymbol{p}\right)}{\sum_{i=1}^{N-1} i^2 p_i}, \tag{18}$$

where $\ell_0 = \max\{\lceil \log_2[(2^{S_e} - 1)/(N-1)] \rceil, 2\}$, $H(\boldsymbol{p}) = -\sum_{i=0}^{N-1} p_i \log_2 p_i$ is the information entropy, and $\boldsymbol{p} = (p_0, p_1, p_2, \ldots, p_{N-1}) \in [0,1]^N$ is the solution of the set of equations:

$$S_e = \ell \sum_{i=1}^{N-1} p_i + H\left(\boldsymbol{p}\right)$$
$$\left(2j+1\right)\ell = j^2 \log_2 \frac{p_0}{p_{j+1}} - \left(j+1\right)^2 \log_2 \frac{p_0}{p_j}, \tag{19}$$

for $j \in \{1, 2, \ldots, N-2\}$. Here, $p_0 = 1 - \sum_{i=1}^{N-1} p_i$ and $\lceil x \rceil$ denotes the minimum integer not less than x.

Proof: The multinomial coefficient can be bounded with the aid of the method of types as follows [14]:

$$2^{KH\left(\frac{\boldsymbol{n}}{K}\right)} \geq \frac{K!}{\prod_{i=1}^{N-1} n_i! \left(K - \sum_{i=1}^{N-1} n_i\right)!} \geq \frac{2^{KH\left(\frac{\boldsymbol{n}}{K}\right)}}{K+1}. \tag{20}$$

Substituting in (8) and (15) yields:

$$\ell \sum_{i=1}^{N-1} p_i + H\left(\boldsymbol{p}\right) \geq \eta \geq \ell \sum_{i=1}^{N-1} p_i + H\left(\boldsymbol{p}\right) - \frac{\log_2(K+1)}{K} \tag{21}$$

and

$$\frac{3/2}{(\kappa_\ell 2^\ell - 1)\sum_{i=1}^{N-1} i^2 p_i}\left[\ell \sum_{i=1}^{N-1} p_i + H(\boldsymbol{p})\right] \geq \gamma \geq \frac{3/2}{(\kappa_\ell 2^\ell - 1)\sum_{i=1}^{N-1} i^2 p_i} \times \left[\ell \sum_{i=1}^{N-1} p_i + H(\boldsymbol{p}) - \frac{\log_2(K+1)}{K}\right], \quad (22)$$

respectively, where $\ell = \log_2 L \in \mathbb{N}^+$, $p_i = n_i/K$, $i \in \{1, 2, \ldots, N-1\}$, and $p_0 = 1 - \sum_{i=1}^{N-1} p_i$. In view of the tightness of the bounds in (21) and (22) for large values of K, we have:

$$\theta_{N\text{MPPM-}L\text{QAM}}(S_e, N) = \max_{\substack{\boldsymbol{p}\in[0,1]^N, \ell\in\mathbb{N}^+: \\ \ell\sum_{i=1}^{N-1} p_i + H(\boldsymbol{p}) \geq S_e, \\ \sum_{i=0}^{N-1} p_i = 1}} \frac{3/2}{\kappa_\ell 2^\ell - 1} \cdot \frac{\ell \sum_{i=1}^{N-1} p_i + H(\boldsymbol{p})}{\sum_{i=1}^{N-1} i^2 p_i}. \quad (23)$$

The constraint function is concave with a maximum of $\log_2[1 + (N-1)2^\ell]$, achieved at $\boldsymbol{p}^*$ with $p_i^* = 2^\ell/[1 + (N-1)2^\ell]$, $i \in \{1, 2, \ldots, N-1\}$. Accordingly, we have:

$$\ell \geq \log_2 \frac{2^{S_e} - 1}{N-1}. \quad (24)$$

Noticing that the denominator is exponential in ℓ while the numerator is linear in it and since $\kappa_\ell = 5/4$ for ℓ odd and 1 for ℓ even, the power efficiency (objective function) decreases with ℓ (for $\ell \geq 2$). However, there are possibilities of an increase in objective function when ℓ increases from odd to even values. Accordingly, $\theta_{N\text{MPPM-}L\text{QAM}}(S_e, N)$ is maximized for $\ell \in \{\ell_0, \ell_0 + 1\}$, where ℓ_0 satisfies:

$$\ell_0 = \max\left\{\left\lceil \log_2\left[\left(2^{S_e} - 1\right)/(N-1)\right]\right\rceil, 2\right\}. \quad (25)$$

■

1) Necessary Condition: For any given $\ell \in \mathbb{N}^+$, the spectral efficiency (constraint function) is concave in the space of $\boldsymbol{p}$. Accordingly, it is increasing in any direction (in this space) that is heading $\boldsymbol{p}^*$. Let the constraint on spectral efficiency is achieved at the set:

$$\mathcal{T} = \left\{\boldsymbol{p} \in [0,1]^N : \eta = S_e\right\}. \quad (26)$$

Given that there exists a direction path, on the set of vectors $\boldsymbol{p}$, over which the power efficiency (objective function) decreases with increasing the spectral efficiency, the constraint on spectral efficiency holds and the objective function is maximized over $\mathcal{T}$. Accordingly, we define a Lagrangian function as follows:

$$\mathcal{L}(\boldsymbol{p}, \lambda) = \frac{\ell \sum_{i=1}^{N-1} p_i + H(\boldsymbol{p})}{\sum_{i=1}^{N-1} i^2 p_i} - \lambda\left[S_e - \ell \sum_{i=1}^{N-1} p_i - H(\boldsymbol{p})\right], \quad (27)$$

where λ is a Lagrange multiplier. Setting the gradient $\nabla\mathcal{L}(\boldsymbol{p}, \lambda)$ to zero yields the set of equations:

$$j^2\left[\ell \sum_{i=1}^{N-1} p_i + H(\boldsymbol{p})\right] = \left(\ell + \log_2 \frac{p_0}{p_j}\right) \times \left[\lambda\left(\sum_{i=1}^{N-1} i^2 p_i\right)^2 + \left(\sum_{i=1}^{N-1} i^2 p_i\right)\right], \quad (28)$$

for $j \in \{1, 2, \ldots, N-1\}$. We eliminate λ from the last equations as follows:

$$\frac{j^2}{i^2} = \frac{\ell + \log_2(p_0/p_j)}{\ell + \log_2(p_0/p_i)}, \quad (29)$$

for any $i, j \in \{1, 2, \ldots, N-1\}$. That is,

$$\left(i^2 - j^2\right)\ell + i^2 \log_2 \frac{p_0}{p_j} - j^2 \log_2 \frac{p_0}{p_i} = 0. \quad (30)$$

Substituting $i = j+1$, $j \in \{1, 2, \ldots, N-2\}$, in the last equation yields the set of equations (19).

2) Sufficient Condition: We explain here that the set of equations (30) determines the optimization path, rewritten as:

$$\mathcal{G} = \left\{\boldsymbol{p} \in [0,1]^N : \left(i^2 - j^2\right)\ell - j^2 \log_2 \frac{p_0}{p_i} + i^2 \log_2 \frac{p_0}{p_j} = 0, \forall i, j \in \{1, 2, \ldots, N-1\}\right\}. \quad (31)$$

It is worth noticing that $\boldsymbol{p}^* \in \mathcal{G}$. First, we get the gradient of the power efficiency:

$$\nabla\gamma = \begin{pmatrix} \frac{\partial\gamma}{\partial p_1} & \frac{\partial\gamma}{\partial p_2} & \cdots & \frac{\partial\gamma}{\partial p_{N-1}} \end{pmatrix}^T, \quad (32)$$

where $(\cdot)^T$ denotes the transpose of a vector. It is easy to show that for any $j \in \{1, 2, \ldots, N-1\}$,

$$\frac{\partial\gamma}{\partial p_j} = \frac{3/2}{(\kappa_\ell 2^\ell - 1)\left(\sum_{i=1}^{N-1} i^2 p_i\right)^2}\left[j^2 \log_2 p_0 + \sum_{i=1}^{N-1} p_i\left(\left(i^2 - j^2\right)\ell + i^2 \log_2 \frac{p_0}{p_j} - j^2 \log_2 \frac{p_0}{p_i}\right)\right], \quad (33)$$

where $p_0 = 1 - \sum_{i=1}^{N-1} p_i$. Now, for any $\boldsymbol{p} \in \mathcal{G}$, the last equation reduces to:

$$\frac{\partial\gamma}{\partial p_j} = \frac{3/2}{(\kappa_\ell 2^\ell - 1)\left(\sum_{i=1}^{N-1} i^2 p_i\right)^2} \cdot j^2 \log_2 p_0 < 0. \quad (34)$$

That is, γ decreases as $\boldsymbol{p}$ increases in $\mathcal{G}$. Since η increases for any direction heading $\boldsymbol{p}^*$, we move in the direction that decreases η so as to maximize γ, till we reach $\eta = S_e$. Hence, the optimizing distribution is $\hat{\boldsymbol{p}} \in \mathcal{G} \cap \mathcal{T}$.

C. 2l.MPPM-LQAM Constrained Power Efficiency

The constrained power efficiency for the special case of 2l.MPPM-LQAM is determined by setting $N = 3$ and $p_2 =$

$1 - p_1$ in (18):

$$\theta_{2l.\text{MPPM-}L\text{QAM}}(S_e) = \frac{3/2}{\kappa_\ell 2^\ell - 1} \cdot \frac{\ell + h(p)}{3p+1}, \tag{35}$$

where $h(p)$ is the binary entropy function, $\ell = \max\{\lceil S_e - 1\rceil, 2\}$, and p is the smaller root of the equation $\ell + h(p) = S_e$.

D. NMPPM Constrained Power Efficiency

The constrained power efficiency of proposed NMPPM is evaluated using a similar analysis to what we did in in the previous section with the following observations:

$$\ell = 0 \quad \text{and} \quad \mathcal{E}_s = \sum_{i=1}^{N-1} \alpha_i^2 p_i. \tag{36}$$

Accordingly, we have:

$$\theta_{N\text{MPPM}}(S_e, N) = \frac{H(\boldsymbol{p})}{\sum_{i=1}^{N-1} i^2 p_i}, \tag{37}$$

where $\boldsymbol{p} = (p_0, p_1, p_2, \ldots, p_{N-1}) \in [0,1]^N$ is the solution of the set of equations (19) with $\ell = 0$. Notice that a solution exists only if $\log_2 N \geq S_e$.

E. NPAM-LQAM Constrained Power Efficiency

The constrained power efficiency of proposed NPAM-LQAM is evaluated using a similar analysis to what we did in the previous section with the following observations:

$$d_{\min} = \sqrt{2} \cdot \min\left\{\sqrt{2}\alpha_1, \min_{i\in\{2,\ldots,N-1\}}\{\alpha_i - \alpha_{i-1}\}\right\},$$
$$\mathcal{E}_s = \frac{2}{3}(\kappa_\ell L - 1)\sum_{i=1}^{N-1}\alpha_i^2 p_i, \tag{38}$$

and $p_i = 1/(N-1), i \in \{1, 2, \ldots, N-1\}$. Selecting the α_i's so that $d_{\min}$ is maximized, we get:

$$\alpha_i = \frac{1 + (i-1)\sqrt{2}}{1 + (N-2)\sqrt{2}}, \qquad i \in \{1, 2, \ldots, N-1\}, \tag{39}$$

and

$$d_{\min} = \frac{2}{1 + (N-2)\sqrt{2}}. \tag{40}$$

Accordingly, we have:

$$\begin{aligned}
&\theta_{N\text{PAM-}L\text{QAM}}(S_e, N) \\
&= \max_{\substack{\ell\in\mathbb{N}^+: \\ \ell+\log_2(N-1)\geq S_e}} \frac{3/2}{\kappa_\ell 2^\ell - 1} \\
&\quad \times \frac{\ell + \log_2(N-1)}{1/(N-1)\sum_{i=1}^{N-1}\left[1 + (i-1)\sqrt{2}\right]^2} \\
&= \frac{3/2}{\kappa_{\ell_0} 2^{\ell_0} - 1} \cdot \frac{\ell_0 + \log_2(N-1)}{1 + (N-2)\left(2N/3 - 1 + \sqrt{2}\right)}, \end{aligned} \tag{41}$$

where $\ell_0 = \max\{\lceil S_e - \log_2(N-1)\rceil, 2\}$ and we used the assertion: $\sum_{i=1}^{n} i^2 = n(n+1)(2n+1)/6$.

1) NPAM Constrained Power Efficiency: It is easy to check that for traditional NPAM scheme, we have:

$$\theta_{N\text{PAM}}(S_e, N) = \frac{\log_2(N-1)}{1 + (N-2)\left(2N/3 - 1 + \sqrt{2}\right)}. \tag{42}$$

V. Numerical Results

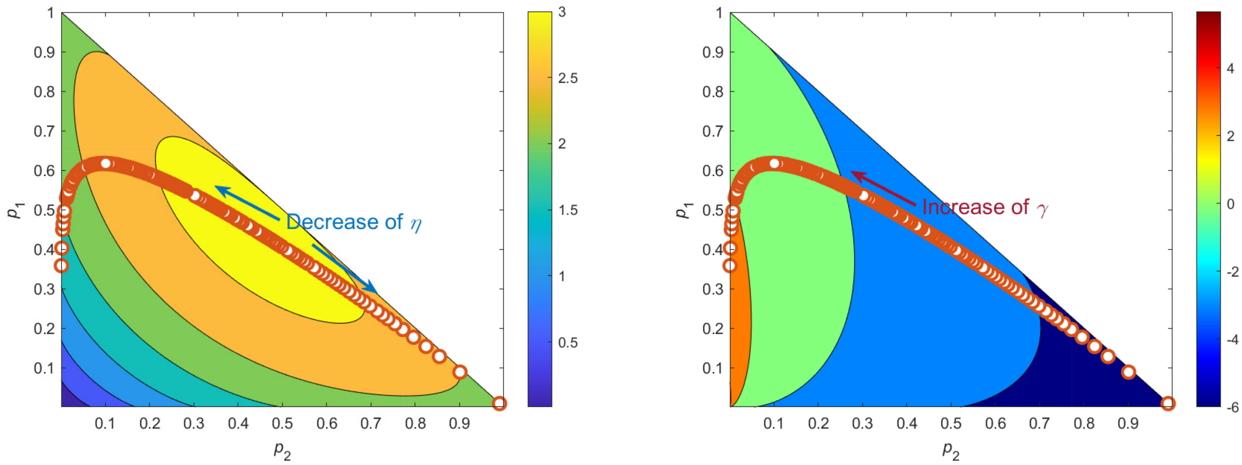
In this section, we give numerical comparisons of the achievable constrained power efficiencies of proposed schemes against traditional techniques for different spectral efficiency constraints.

A. Contour Plots of NMPPM-LQAM Efficiencies

We start in Fig. 2 by plotting the contours for both spectral and power efficiencies of proposed NMPPM-LQAM technique with $N = 3$. The direction path, as determined by (31), is shown in same figures as well for the sake of illustration. It is clear that a direction exists along this path where the power efficiency increases with a decrease in the spectral efficiency. Following this direction leads to the best achievable power efficiency for a given spectral efficiency constraint.

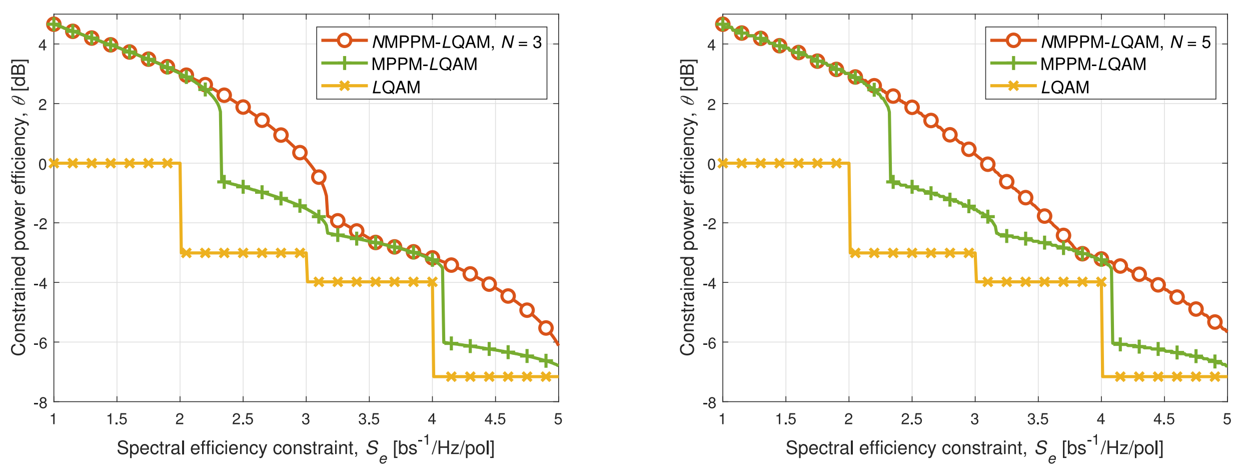
B. Constrained Power Efficiency of NMPPM-LQAM Scheme

Figure 3 shows comparisons between the constrained power efficiencies of proposed NMPPM-LQAM scheme against traditional MPPM-LQAM and LQAM techniques for a range of spectral efficiency thresholds. Figure 3(a) shows such a comparison for $N = 3$, while Fig. 3(b) shows it for $N = 5$. The sharp jumps in both traditional LQAM and MPPM-LQAM curves signify rises in the QAM levels so as to fulfill the restraints on the spectral efficiencies. For example, in traditional LQAM, if $S_e \leq 2$ bs^{-1}/Hz/pol, $L = 2^2 = 4$ gives the optimum power efficiency. Increasing S_e above 2 dictates L to increase to $2^3 = 8$ so as to achieve the spectral efficiency constraint. Two different step sizes appear when switching from even ℓ to odd ℓ or vice versa. These jumps indicate that significant penalties are to be paid in transmitted power for increasing the transmission rate. For example, if the spectral efficiency is to increase slightly from 2 to 2.1 bs^{-1}/Hz/pol, a penalty of about 3 dB in the power efficiency of QAM is to be paid. On the other hand, it is clear from the figures that these penalties have been relaxed in proposed scheme, which provides gradual decrease in the power efficiency as S_e increases. Specifically, the improvements in power efficiency when using proposed system at $S_e = 2.5$ bs^{-1}/Hz/pol are about 4.87 and 2.67 dB when compared to that of traditional LQAM and traditional MPPM-LQAM techniques, respectively. It is also clear that increasing the MPPM level would increase the power efficiency even better as seen from Fig. 3(b). Indeed, the gradual decrease in power efficiency is extended over a broader range of S_e. Specifically at $S_e = 3.5$ bs^{-1}/Hz/pol, both NMPPM-LQAM (with $N = 3$) and traditional MPPM-LQAM schemes have same constrained power efficiencies, while NMPPM-LQAM (with $N = 5$) has an improved power efficiency. It is remarkable to notice that as the spectral efficiency constraint S_e increases from 1 to 3.82 bs^{-1}/Hz/pol, the QAM levels of both traditional MPPM-LQAM and LQAM increases



(a) (b)

Fig. 2. Contour plots for both (a) spectral and (b) power efficiencies of proposed NMPPM-LQAM scheme with $N = 3$. The direction path where the power efficiency increases with decreasing the spectral efficiency is shown with circle markers.



(a) (b)

Fig. 3. Comparisons between constrained power efficiencies of NMPPM-LQAM scheme against traditional MPPM-LQAM and LQAM formats versus spectral efficiency restraints for: (a) $N = 3$; (b) $N = 5$.

three times $L \in \{4, 8, 16\}$. However, the corresponding QAM level in proposed NMPPM-LQAM (with $N = 5$) is retained at the lowest level ($L = 4$) over the same range of S_e.

C. Constrained Power Efficiencies of NMPPM, $2l$.MPPM-LQAM, and NPAM-LQAM Techniques

Finally, in Fig. 4 we give a comparison for the constrained power efficiencies of NMPPM, $2l$.MPPM-LQAM, and NPAM-LQAM against traditional MPPM-LQAM and LQAM schemes. The figure indicates that the constrained power efficiency of $2l$.MPPM-LQAM technique is better than that of traditional QAM at all S_e levels. Improvement of more than 3 dB is achievable at specific levels. In addition, the sharp jumps due to QAM level increase are relaxed. However, traditional MPPM-LQAM scheme is competitive to $2l$.MPPM-LQAM. Specifically, at low spectral efficiency, e.g., $S_e = 2$ bs^{-1}/Hz/pol, traditional MPPM-LQAM shows better power efficiency, while for high spectral efficiency, e.g., $S_e = 4.5$ bs^{-1}/Hz/pol, the inverse is true. Compared to Fig. 3, it is clear that proposed NMPPM-LQAM format is better than $2l$.MPPM-LQAM over the entire range range of S_e. Indeed, in NMPPM-LQAM scheme, the power efficiency is optimized.

As for proposed NMPPM and NPAM-LQAM techniques, the figure illustrates that the former is power efficient over a small spectral range, while the latter has a low constrained power efficiency compared to traditional QAM.

It can be concluded that combining NMPPM with LQAM provides a superior technique that is both power and spectral efficient simultaneously. On the other hand, combining NPAM with LQAM does not provide an improvement over LQAM. This is because the PAM scheme is non-power efficient. However, NPAM-LQAM is more spectral and power efficient than traditional NPAM (plotted in same figure as well). Indeed, the latter has a maximum spectral efficiency of $\log_2(N-1)$, while the former has a higher spectral efficiency of $\ell + \log_2(N-1)$.

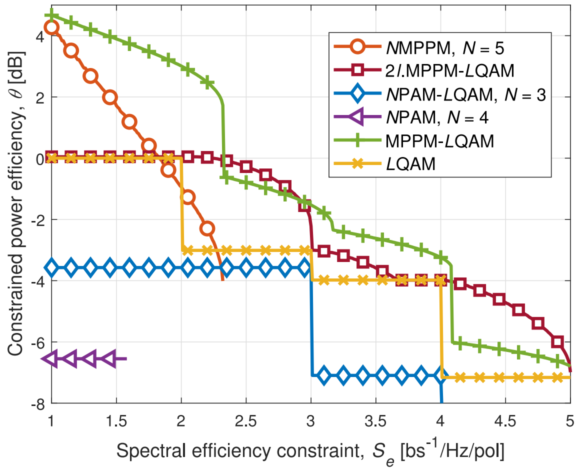


Fig. 4. Comparisons among constrained power efficiencies of NMPPM, $2l$.MPPM-LQAM, and NPAM-LQAM schemes against traditional MPPM-LQAM and LQAM techniques for various spectral efficiency restraints.

VI. Implementation Complexity

Here, we give a note on the implementation complexity of proposed NMPPM-LQAM system. As the system combines both QAM and N level MPPM, the receiver should be able to detect the levels in addition to the relative positioning. The detection of the MPPM levels can follow the detection of the leveling in traditional PAM formats. The detection of relative positioning and QAM leveling is similar to that of traditional MPPM-LQAM system. In fact, it is even simpler. Indeed, the QAM level here is very low compared to that of traditional LQAM and MPPM-LQAM systems, cf. Fig. 3(b).

VII. Conclusion

A new power efficient technique, called N-level multipulse PPM-LQAM and denoted by NMPPM-LQAM, has been proposed. Its constrained power efficiency has been derived, characterized, and evaluated. It has been shown that combining NMPPM with LQAM provides a superior technique that is both power and spectral efficient simultaneously. Alterations to this technique, that comprise NMPPM, $2l$.MPPM-LQAM, and NPAM-LQAM schemes, have been considered and characterized as well. Comparisons with well-known formats have been performed numerically for a range of spectral efficiency thresholds. The results divulge that proposed NMPPM-LQAM is the most power/spectral efficient technique.

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Hossam M. H. Shalaby (Senior Member, IEEE) was born in Giza, Egypt, in 1961. He received the B.S. and M.S. degrees from Alexandria University, Alexandria, Egypt, in 1983 and 1986, respectively and the Ph.D. degree from the University of Maryland at College Park in 1991, all in electrical engineering. In 1991, he joined the Electrical Engineering Department at Alexandria University, and was promoted to a Professor in 2001. Since 2017, he has been on leave from Alexandria University, where he is a Professor with the Department of Electronics and Communications Engineering (ECE), School of Electronics, Communications, and Computer Engineering, Egypt-Japan University of Science and Technology (E-JUST), New Borg EL-Arab City, Alexandria, Egypt. From 2010 to 2016, he was the Chair of the ECE Department, E-JUST. From December 2000 to 2004, he was an Adjunct Professor with the Faculty of Sciences and Engineering, Department of Electrical and Information Engineering, Laval University, Quebec, QC, Canada. From 1996 to 1998, he was with the Electrical and Computer Engineering Department, International Islamic University Malaysia and from 1998 to 2001, he was with the School of Electrical and Electronic Engineering, Nanyang Technological University, Singapore. He worked as a Consultant at SysDSoft company, Alexandria, Egypt, from 2007 to 2010. His research interests include optical communications, silicon photonics, optical CDMA, and quantum information theory. He serves as a Student Branch Counselor of the IEEE E-JUST student branch since 2020. He served as a Student Branch Counselor at Alexandria University, IEEE Alexandria, and North Delta Subsection, from 2002 to 2006, and served as a Chairman of the Student Activities Committee of IEEE Alexandria Subsection from 1995 to 1996. He received an SRC fellowship from 1987 to 1991 from the Systems Research Center, Maryland; State Excellence Award in Engineering Sciences in 2007 from the Academy of Scientific Research and Technology, Egypt; Shoman Prize for Young Arab Researchers in 2002 from the Abdul Hameed Shoman Foundation, Amman, Jordan; State Incentive Award in Engineering Sciences in 1995 and 2001 from the Academy of Scientific Research and Technology, Egypt; University Excellence Award in 2009 from Alexandria University; and University Incentive Award in 1996 from Alexandria University. He is a Senior Member of the IEEE Photonics Society and the Optical Society (OSA).