Optics Communications 356 (2015) 530–535

Contents lists available at ScienceDirect

Optics Communications

journal homepage: www.elsevier.com/locate/optcom

SER Analysis of MPPM-Coded MIMO-FSO System over Uncorrelated and Correlated Gamma-Gamma Atmospheric Turbulence Channels

Haitham S. Khallaf [a], José M. Garrido-Balsells [b], Hossam M.H. Shalaby [a,c], Seiichi Sampei [d]

[a] Egypt-Japan University of Science and Technology (E-JUST), Alexandria 21934, Egypt
[b] University of Málaga, 29071 Málaga, Spain
[c] Alexandria University, Alexandria 21544, Egypt
[d] Graduate School of Engineering, Osaka University, 2-1 Yamada-oka, Suita, Osaka 565-0871, Japan

ARTICLE INFO

Article history:
Received 29 June 2015
Received in revised form
19 August 2015
Accepted 20 August 2015

Keywords:
Free-space optics
gamma-gamma channels
Multiple-input multiple-output (MIMO)
Multi-pulse pulse-position modulation (MPPM)
Shot-noise limited receivers
$\alpha-\mu$ distribution

ABSTRACT

The performance of multiple-input multiple-output free space optical (MIMO-FSO) communication systems, that adopt multipulse pulse position modulation (MPPM) techniques, is analyzed. Both exact and approximate symbol-error rates (SERs) are derived for both cases of uncorrelated and correlated channels. The effects of background noise, receiver shot-noise, and atmospheric turbulence are taken into consideration in our analysis. The random fluctuations of the received optical irradiance, produced by the atmospheric turbulence, is modeled by the widely used gamma-gamma statistical distribution. Uncorrelated MIMO channels are modeled by the $\alpha-\mu$ distribution. A closed-form expression for the probability density function of the optical received irradiance is derived for the case of correlated MIMO channels. Using our analytical expressions, the degradation of the system performance with the increment of the correlation coefficients between MIMO channels is corroborated.

1. Introduction

Free-space optical (FSO) communications systems have high bandwidth, no spectrum licensing, and interference immunity. They provide cost-effective means for transferring high data rates and are used in many telecommunications applications, e.g., satellite links, robotics, last mile connectivity, cellular backhaul, and optical-fiber backup [1]. One of the most important phenomena that affect the performance of FSO system is Scintillation [2,3]. Scintillation is produced by the inhomogeneities of the temperature and pressure along the transmission path, which cause random fluctuations on the refractive index seen by the optical signal propagating along the FSO link, which in turn cause random fluctuations in both amplitude and phase of the received optical field. This fluctuations increase the system symbol error rate (SER). Several statistical distributions have been proposed to model the random effects of the turbulence induced scintillation, e.g., log-normal, negative-exponential, and gamma-gamma distributions. Log-normal distributions are only suitable for weak turbulence conditions, while negative-exponential distributions are suitable for very strong or extreme turbulence conditions [4]. Gamma-gamma distributions, however, are valid for a wider range of turbulence intensities from weak to strong [5]. The last model is used in this paper to describe the random intensity fluctuations generated by scintillation.

The effect of scintillation can be reduced by using spatial diversity where there are multiple apertures at the transmitter and/or the receiver sides. In this paper, the most complete spatial diversity solution, i.e., multiple-input-multiple-output (MIMO) technique, is employed [6]. Additionally, in order to increase the system robustness against the scintillation effects, multipulse pulse-position modulation (MPPM) is adopted. MPPM has been introduced in [7], where multiple signal slots are sent during a symbol duration. Thus, MPPM has better bandwidth and power efficiencies when compared to standard pulse-position modulation (PPM) and on-off-Keying (OOK) schemes, respectively.

The performance of MIMO-FSO systems has been widely investigated in literature. Wilson et al. have studied the performance of MIMO-FSO system using MPPM coding methods under both log-normal and Rayleigh fading models [6], Navidpour et al. have investigated the bit-error rate (BER) performance of a MIMO-FSO system adopting OOK over log-normal turbulent fading channels for both cases of independent and correlated channels [8], Kazemi et al. have studied the outage probability of MIMO-FSO system with selection combining (SC) over log-normal turbulent channels [9], Cvijetic et al. have derived closed-form bounds for a MIMO-FSO system adopting PPM for non-fading, log-normal, and

E-mail address: eng.h.khallaf@gmail.com (H.S. Khallaf).

http://dx.doi.org/10.1016/j.optcom.2015.08.060
0030-4018/

negative exponential fading channels [10] and Jiang et al. have investigate the BER performance of MIMO-FSO adopting OOK under turbulent channel modelled by exponentiated Weibull (EW) [11]. The previously maintained literatures have studied the effects of fading for different modulation techniques but haven't use the gamma-gamma pdf, which is the most valid one, to model scintillation. In [12,13] the gamma-gamma distribution has been used to model turbulent channel but the gamma-gamma summation of MIMO channels has been approximated with gamma-gamma and gamma distribution, respectively. Finally, Kazemi et al. have derived a closed-form expression for the outage probability for a MIMO-FSO system adopting binary PPM [14]. They have used a α-μ distribution to model the atmospheric channel random effects. Their work can be considered as a special case of our work since they study the performance of PPM in case of uncorrelated channel while we study the common case of MPPM in case of uncorrelated and correlated channels.

The main contribution in this paper is to derive closed-form expressions for the SERs of MIMO-FSO systems employing MPPM coding techniques under gamma-gamma distributed atmospheric turbulent fluctuations. In our derivations, we use an equal gain combining (EGC) method and different correlation characteristics between the received optical signals. For uncorrelated (independent) channels, the resultant combined summation of the gamma-gamma received signals can be approximated by the α-μ distribution [15]. For the general case of correlated channel, the summation is obtained by applying a recently proposed model, which defines the probability-density function (pdf) of a combined received optical irradiance when correlated spatial diversity is employed at the receiver (single-input-multiple-output (SIMO)) [16]. Moreover, in order to reduce the computing complexity of the exact expressions, approximate closed-form expressions for the SERs are also derived.

The rest of the paper is organized as follows. Gamma-gamma channel model is cited in Section 2. In Section 3, our MIMO-FSO system and channel models are introduced, where both uncorrelated and correlated channels are considered. Section 4 is devoted for the derivation of exact and approximate closed-form expressions of the SERs in both cases of uncorrelated and correlated MIMO channels. Our numerical results are given in Section 5, where we make comparisons between both SISO and MIMO systems using both exact and approximate expressions. The effect of correlation coefficients on the system performance is investigated in this section as well. Finally, the conclusion is given in Section 6.

2. Gamma-gamma channel models

The marginal pdf of the gamma-gamma distribution is defined as [5]:

$$g(K_s) = \frac{2(xy)^{\frac{x+y}{2}}}{\Omega\Gamma(x)\Gamma(y)}\left(\frac{K_s}{\Omega}\right)^{\frac{x+y}{2}-1} \times B_{x-y}\left(2\sqrt{\frac{xyK_s}{\Omega}}\right), \quad K_s \ge 0, \tag{1}$$

where $\Gamma(\cdot)$ denotes the gamma function, $B_c(\cdot)$ denotes the c_{th} order modified Bessel function of the second kind, K_s represents the average detected signal photon count per signal slot, and $\Omega = E\{K_s\}$ is the mean of K_s. Here $E\{\cdot\}$ denotes the expected value. The parameters x and y are the shape parameters of the gamma-gamma distribution, associated to refractive and diffractive turbulence effects, respectively. They are related to the effective number of large-scale and small-scale eddies, respectively. The scintillation index $\chi_{SC}^2 \overset{\text{def}}{=} var\{K_s\}/E\{K_s^2\}$, which provides a measure to the strength of the intensity fluctuations, is related to x and y as:

$$\chi_{SC}^2 = \frac{1}{x} + \frac{1}{y} + \frac{1}{xy}. \tag{2}$$

For weak to moderate turbulence, $\chi_{SC}{}^2$ is usually in the range of $[0,\ 0.75]$, whereas for strong turbulence, the scintillation index is greater than 0.75 [4].

3. MIMO-FSO system and channel models

Our MIMO-FSO system is composed of M transmit and N receive apertures as shown in Fig. 1.

The transmitters employ MPPM techniques and the receivers are assumed to be shot-noise limited. An MPPM symbol duration is divided into Q time slots and the optical power is transmitted within $w \in \{1, 2, \ldots, Q/2\}$ time slots only. Number of bits per frame in MPPM modualtion technique is $\log_2\binom{Q}{w}$. The same MPPM frame will be transmitted by all the M laser sources. Let Y_{mn}^j, $m \in \{1, 2, \ldots, M\}$, $n \in \{1, 2, \ldots, N\}$, denote the detected photon count in slot $j \in \{1, 2, \ldots, Q\}$ over the optical link established between mth transmitter and nth receiver. The average detected photon count per signal slot for EGC-based receiver is given by

$$K_{s_{on}} = \sum_{m=1}^{M}\sum_{n=1}^{N} K_{s_{mn}} + K_b, \tag{3}$$

where $K_{s_{mn}}$ denotes the average detected signal photon count over the optical link between mth transmitter and nth receiver and K_b is the average detected photon count per slot due to background noise. In order to evaluate the average SER of the system, the pdf of any of the Q summations of the MN gamma-gamma random variables

$$Z^j = \sum_{m=1}^{M}\sum_{n=1}^{N} Y_{mn}^j, \quad j \in \{1, 2, \ldots, Q\}, \tag{4}$$

should be defined. This pdf is dependent on the relationship between the MIMO channels. In our analysis below we consider two cases, uncorrelated (independent) and correlated (more realistic) MIMO channels.

3.1. Uncorrelated MIMO channels

If the distance between any two elements of the system (M laser sources and N photodetectors) is greater than the correlation distance ρ_c, each of the individual paths from transmitter to receiver can be considered to be independent. The correlation

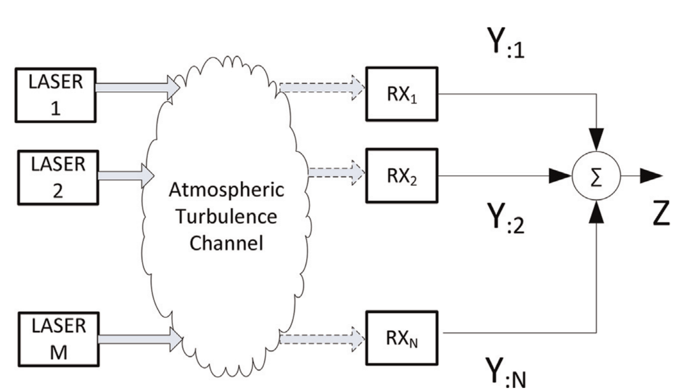

Fig. 1. Multiple-input multiple-output (MIMO) system model with M laser transmitters and N optical receivers.

distance ρ_c is typically approximated by $\sqrt{\lambda L}$ for weak turbulence, where λ is the wavelength and L is the distance between transmitter and receiver [17]. In this case, Z is a summation of MN independent and identically distributed (i.i.d) gamma-gamma random variables. This summation can be approximated using α-μ distribution, which probability-distribution functions are given by [15,18]:

$$f_Z(z) = \frac{\alpha\mu^{\mu} z^{\alpha\mu-1}}{\hat{z}^{\alpha\mu}\Gamma(\mu)} \exp\left(-\mu\frac{z^{\alpha}}{\hat{z}^{\alpha}}\right), \tag{5}$$

where α, $\mu > 0$ are the distribution parameters and $\hat{z} = \sqrt[\alpha]{E\{z^{\alpha}\}}$ is an α-root mean value. To calculate α, μ, and $\hat{z}$, the moment-based parameter estimators introduced in [15,18] are employed, leading to solving the following nonlinear equations:

$$\begin{aligned}
&\frac{\Gamma^2(\mu+\frac{1}{\alpha})}{\Gamma(\mu)\Gamma(\mu+\frac{2}{\alpha}) - \Gamma^2(\mu+\frac{1}{\alpha})} = \frac{E^2\left\{Z\right\}}{E\left\{Z^2\right\} - E^2\left\{Z\right\}}\\
&\frac{\Gamma^2(\mu+\frac{2}{\alpha})}{\Gamma(\mu)\Gamma(\mu+\frac{4}{\alpha}) - \Gamma^2(\mu+\frac{2}{\alpha})} = \frac{E^2\left\{Z^2\right\}}{E\left\{Z^4\right\} - E^2\left\{Z^2\right\}}\\
&\hat{z} = \frac{\mu^{\frac{1}{\alpha}}\Gamma(\mu)E\left\{Z\right\}}{\Gamma\left(\mu+\dfrac{1}{\alpha}\right)}.
\end{aligned} \tag{6}$$

By using the multinomial expansion theorem, the required moments $E\{Z\}$, $E\{Z^2\}$, and $E\{Z^4\}$ can be evaluated as follows:

$$E\{Z^{\nu}\} = \sum_{j_1=0}^{\nu}\sum_{j_2=0}^{j_1}\cdots\sum_{j_{MN-1}=0}^{j_{MN-2}} \binom{\nu}{j_1}\binom{j_1}{j_2}\cdots\binom{j_{MN-2}}{j_{MN-1}} \times E\{L_1^{\nu-j_1}\}E\{L_2^{j_1-j_2}\}\cdots E\{L_{MN}^{j_{MN-1}}\}, \tag{7}$$

where $L_i = K_{smn}$ for $i = (m-1)N + n$, and ν is a positive integer. The νth moments of L_i can be computed as

$$E\{L_i^{\nu}\} = \left(\frac{xy}{\Omega}\right)^{-\nu}\frac{\Gamma(x+\nu)\Gamma(y+\nu)}{\Gamma(x)\Gamma(y)}. \tag{8}$$

3.2. Correlated MIMO channels

Due to the limitation in physical space, it may be difficult to keep the spacing between receivers greater than the correlation distance. In these situations, the MIMO-FSO channels exhibit spatial correlation and the α-μ distribution is no longer valid for the random variable Z of (4). A closed-form expression for the pdf of the summation of EGC correlated gamma-gamma random variables has been introduced by Garrido-Balsells et al. [16]. In that paper, the pdf of the combined received irradiance is deduced in the case of a SIMO-FSO system. In their analysis, the authors have assumed that under the most usual atmospheric turbulence conditions, the large-scale effects are common to all received beams whereas the small-scale effects are described by identically distributed spatially correlated gamma random variables. Applying same assumptions here, we can obtain a generalized closed-form expression for the pdf of the EGC combined optical irradiance in a MIMO system configuration:

$$f_Z(z) = \frac{2}{MN\Omega\left[\det(A)\right]^{y}\Gamma(x)}\sum_{i=1}^{N'}\sum_{q=1}^{y_i}\frac{c_{qi}\lambda_i^{\frac{q-x}{2}}}{\Gamma(q)} \times x^{\frac{q+x}{2}}\left(\frac{z}{MN\Omega}\right)^{MNy-1-\frac{q-x}{2}} B_{q-x}\left(2\sqrt{\frac{xz}{MN\Omega\lambda_i}}\right), \tag{9}$$

where $\{\lambda_i\}_{i=1}^{MN}$ are the eigenvalues of the matrix $\mathbf{A} = \mathbf{DC}$. The matrix $\mathbf{D}$ is an $MN \times MN$ diagonal matrix with the entries of the small-scale shape parameter y, whereas $\mathbf{C}$ is an $MN \times MN$ positive definite correlation matrix whose elements, $\sqrt{\rho_{ij}}$, are the correlation coefficients of the underlying Gaussian processes that lead to the small-scale gamma fading. The upper summation limit N' is the number of different eigenvalues of the matrix $\mathbf{A}$ and $y_i = \mu_A(\lambda_i)y$, where $\mu_A(\lambda_i)$ is the algebraic multiplicity of the eigenvalue λ_i. Finally, the coefficient c_{qi} is given by:

$$c_{qi} = \sum_{k_1+\underset{\hat{i}}{\cdots}+k_{N'}=y_i-q} \frac{\binom{y_i-q}{k_1,\underset{\hat{i}}{\cdots},k_{N'}}}{(y_i-q)!}\prod_{j=1,j\neq i}^{N'}\left[(-1)^{k_j}(y_j)_{k_j}\left(d_j-d_i\right)^{-(y_j+k_j)}\right] \tag{10}$$

where $d_j = \frac{z}{MN\Omega\lambda_j}$, $\binom{y_i-q}{k_1,\underset{\hat{i}}{\cdots},k_{N'}} = \frac{(y_i-q)!}{k_1!,\underset{\hat{i}}{\cdots},k_{N'}!}$ means that k_i is omitted from the sequence, and $(y_j)_{k_j} = \frac{\Gamma(y_j+k_j)}{\Gamma(y_j)}$ is the Pochhammer symbol. For this model, it is not valid to be reduced to the uncorrelated case by using identity correlation matrix,$\mathbf{C}$, because this distribution is based on the assumption that all beams have the same large scale effect so they are already correlated.

4. Symbol-error rates

Let us define the received vector $Z \overset{\text{def}}{=} (Z^1, Z^2, \ldots, Z^Q)$, where the jth entry in this vector Z^j represents a summation over slot j as given in (4). Out of these Q slots, there are w 'ON' slots that carry signal and $Q-w$ 'OFF' slots that carry no signal. The error takes place if one or more of the 'OFF' slots has count equal to or higher than that in 'ON' slots. The SER of MPPM systems in non-turbulent atmosphere channel is given by [19]:

$$SER = \sum_{z_{min}=0}^{\infty}\sum_{l=1}^{Q-w}\sum_{m=1}^{w}\binom{w}{m}\binom{Q-w}{l}p_1\left(z_{min}\right)^m\left(1-P_1\left(z_{min}\right)\right)^{w-m} \times P_0\left(z_{min}-1\right)^{Q-w-l}\left[\left(1-P_0\left(z_{min}\right)\right)^l + p_0\left(z_{min}\right)^l\left(1-\frac{1}{\binom{l+m}{m}}\right)\right] \tag{11}$$

where z_{min} denotes the minimum photon count in symbol signal slots. In addition $p_0(\cdot)$ and $p_1(\cdot)$ denote the photon count probabilities of non-signal and signal slots, respectively. Also, $P_0(\cdot)$ and $P_1(\cdot)$ denote their cumulative distributions. Since the detected photon count per MPPM slot follows a Poisson distribution, the last four probabilities are given by:

$$\begin{aligned}
&p_0(k) = \frac{K_b^k}{k!}e^{-K_b}, \quad p_1(k) = \frac{\left(z+K_b\right)^k}{k!}e^{-(z+K_b)},\\
&P_0(k) = \sum_{j=0}^{k}\frac{K_b^j}{j!}e^{-K_b}, \quad P_1(k) = \sum_{j=0}^{k}\frac{\left(z+K_b\right)^j}{j!}e^{-(z+K_b)},
\end{aligned} \tag{12}$$

for any $k \in \{0, 1, 2, \ldots\}$, where z represents the average detected signal photon count per 'ON' slot. The average SER can be found by averaging (11) with respect to z. Since the term $p_1(z_{min})^m(1-P_1(z_{min}))^{w-m}$ in (11) is the only one that depends on the channel distribution, the average SER is obtained by replacing this term by its average $P_2(z_{min})$ in (11):

$$P_2(z_{min}) = \int_0^{\infty} p_1(z_{min})^m\left(1-P_1\left(z_{min}\right)\right)^{w-m} f_Z(z)dz \tag{13}$$

which, after some algebraic manipulations similar to that in [19], can be expressed as:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \binom{j+mz_{min}}{B} r(j) \times \frac{e^{-wK_b} K_b^{j+mz_{min}-B}}{z_{min}!^m} \int_0^{\infty} z^B e^{-wz} f_Z(z) dz, \tag{14}$$

where $r(j)$, for any integer $j \ge (w-m)(z_{min}+1)$, is defined as the summation $r(j) = \sum_{(s_1,s_2,\ldots,s_{w-m}) \in \mathcal{X}(j)} \frac{1}{s_1! s_2! \cdots s_{w-m}!}$ over the set of vectors $\mathcal{X}(j)$, where

$$\mathcal{X}(j) \overset{\text{def}}{=} \{(s_1, s_2, \ldots, s_{w-m}) \in \mathbb{N}^{w-m}: \sum_{i=1}^{w-m} s_i = j \text{ and } \forall \ell \in \{1, 2, \ldots, w-m\}, z_{min}+1 \le s_\ell \le j-(w-m-1)(z_{min}+1)\}.$$

4.1. Exact SER Expression for Uncorrelated MIMO Channels

Using (5) and (14):

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \binom{j+mz_{min}}{B} r(j) \frac{\alpha\mu^{\mu} e^{-wK_b} K_b^{j+mz_{min}-B}}{z_{min}!^m \Gamma(\mu) \hat{z}^{\alpha\mu}} \int_0^{\infty} z^{\alpha\mu+B-1} e^{-wz} e^{-\frac{\mu z^{\alpha}}{\hat{z}^{\alpha}}} dz. \tag{15}$$

Replacing the two exponential terms in the last integration using Meijer-G function [20] and assuming that α is a rational number $\alpha = t/v \in \mathbb{Q}$, we can use the main integrations formulae in [20] to get the required expression as following:

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \frac{\binom{j+mz_{min}}{B} r(j) \alpha\mu^{\mu} v^{0.5} t^{\alpha\mu+B-\frac{1}{2}}}{e^{wK_b} z_{min}!^m \Gamma(\mu) \hat{z}^{\alpha\mu} (2\pi)^{\frac{t+v-2}{2}}} \times K_b^{j+mz_{min}-B} w^{-(\alpha\mu+B)} G_{t,v}^{v,t}\left(\left(\frac{\mu}{\hat{z}^{\alpha} v}\right)^{v} \left(\frac{t}{w}\right)^{t} \middle| \begin{matrix} \frac{1-\alpha\mu-B}{t}, \ldots, \frac{t-\alpha\mu-B}{t} \\ 0, \frac{1}{v}, \ldots, \frac{v-1}{v} \end{matrix} \right) \tag{16}$$

4.2. Exact SER Expression for Correlated MIMO Channels

By substituting using (9) and (10) into (14), then using the integration formula in [21], the average SER of correlated MIMO-FSO channels reduces to that in (17)

$$P_2(z_{min}) = \sum_{j=(w-m)(z_{min}+1)}^{\infty} \sum_{B=0}^{j+mz_{min}} \sum_{i=1}^{N'} \sum_{q=1}^{y_i} \sum_{k_1+\overset{\cdot\cdot}{\underset{\hat{i}}{\cdot}}+k_{N'}=y_i-q} \binom{y_i-q}{k_1, \overset{\cdot\cdot}{\underset{\hat{i}}{\cdot}}, k_{N'}} \times \binom{j+mz_{min}}{B} \exp\left(\frac{x}{2MN\Omega\lambda_i w} - wK_b\right) w^{-(B+MNy+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2})}$$
$$\times \frac{\Gamma(B+MNy+x-\sum_{j=1}^{N'} y_j)\lambda_i^{\frac{q-x+1}{2}} x^{\frac{q+x-1}{2}} K_b^{j+mz_{min}-B}}{(y_i-q)! z_{min}!^m (MN\Omega)^{\left(MNy+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2}\right)}} \times \frac{r(j)\Gamma(B+MNy+q-\sum_{j=1}^{N'} y_j)}{[\det(A)]^y \Gamma(q)\Gamma(x)}$$
$$\times \prod_{j=1, j\ne i}^{N'} \left[(-1)^{k_j} (y_j)_{k_j} \left(\frac{1}{\lambda_j} - \frac{1}{\lambda_i}\right)^{-(y_j+k_j)} \right] \times W_{-\left(B+MNy+\frac{q+x}{2}-\sum_{j=1}^{N'} y_j-\frac{1}{2}\right), \left(\frac{q-x}{2}\right)}\left(\frac{x}{MN\Omega\lambda_i w}\right). \tag{17}$$

where $W_{(\cdot,\cdot)}(\cdot)$ is the Whittaker function.

4.3. Approximate SER

Due to the computational complexity needed to evaluate both of the exact expressions, we introduce in this subsection an accurate and easily computable approximation expression to obtain an approximate estimations of the SERs. The approximation is based on Gauss-Laguerre quadrature rule [22]. We rewrite (13) as:

$$P_2(z_{min}) = \frac{e^{-(wK_b)}}{z_{min}!^m} \int_0^{\infty} e^{-(wz)} (z+K_b)^{mz_{min}} \times \left[e^{(z+K_b)} - \sum_{j=0}^{z_{min}} \frac{(z+K_b)^j}{j!} \right]^{w-m} f_z(z) dz. \tag{18}$$

Based on Gauss-Laguerre quadrature rule, the value of the integration in (18) can be approximated as:

$$P_2(z_{min}) \approx \sum_{i=1}^{c} \Lambda_i \frac{e^{-wK_b}}{z_{min}!^m} (V_i+K_b)^{mz_{min}} \left[e^{(V_i+K_b)} - \sum_{j=0}^{z_{min}} \frac{(V_i+K_b)^j}{j!} \right]^{w-m} f_Z(V_i), \tag{19}$$

where V_i is the i-th root of the Laguerre polynomial $L_c(x)$ with degree $c > 1$ and Λ_i is the corresponding weighting coefficient.

5. Numerical results

In this section, we investigate the SER performance of MIMO-FSO systems, using both exact and approximate expressions, with Laguerre polynomial with degree c=100, obtained in previous

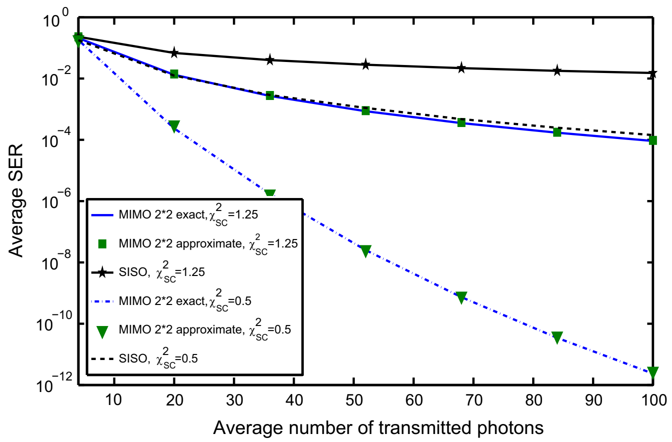

Fig. 2. Average SER versus average number of transmitted photons for an MPPM system with $Q=4$ and $w=2$ under strong and moderate turbulent channel ($\chi_{SC}^2 = 1.25$ and $\chi_{SC}^2 = 0.5$, respectively) for both uncorrelated MIMO (with $M = N = 2$) and SISO systems.

sections. We compare our results to that of the equivalent SISO-FSO system assuming same average transmitted power, data rates, and channel states.

Figs. 2 and 3 show the average SERs versus average number of transmitted photons of MIMO-FSO (with $M = N = 2$) and SISO-FSO systems, both adopting MPPM techniques with $(Q, w) = (4, 2)$. Two levels of turbulence intensity are assumed, strong turbulent channels with parameters ($x=8$, $y=1$, $\chi_{SC}^2 = 1.25$) and moderate turbulent channels with parameters ($x=5$, $y=4$, $\chi_{SC}^2 = 0.5$). Same background noise of $K_b=1$ photon per channel is assumed as well. Specifically, Fig. 2 shows the results for the uncorrelated channels case, whereas Fig. 3 shows the results for correlated MIMO channels with correlation coefficient $\rho = 0.6$. Both exact and approximate expressions are depicted in the two figures, where it is clear that there is a high degree of agreement between their values. It is also clear from the figures that there is a significant improvement in the SER when using MIMO systems when compare to that of the SISO systems. Comparing both figures, we can conclude that uncorrelated MIMO system outperforms the correlated one. The first has SER$=10^{-11}$ under moderate turbulent channels and average number of transmitted photons$=100$ while correlated one has SER$=10^{-4}$ under the same conditions.

Using the same system parameters as give above, Fig. 4 shows

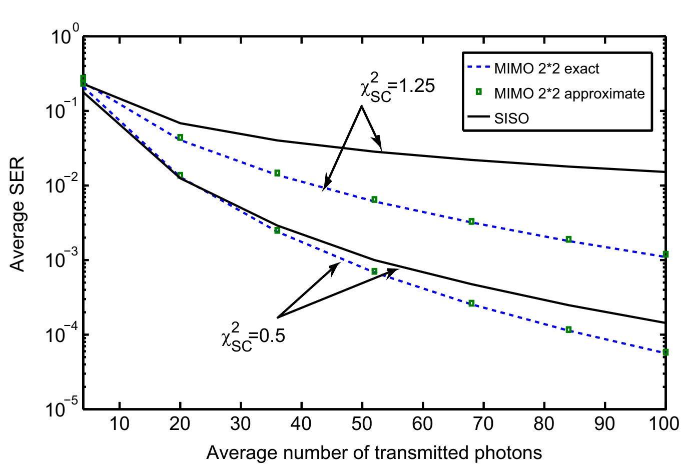

Fig. 3. Average SER versus average number of transmitted photons for an MPPM system with $Q=4$ and $w=2$ under strong and moderate turbulent channel ($\chi_{SC}^2 = 1.25$ and $\chi_{SC}^2 = 0.5$, respectively) for both correlated MIMO, (with $M = N = 2$, correlation coefficient $\rho = 0.6$) and SISO systems.

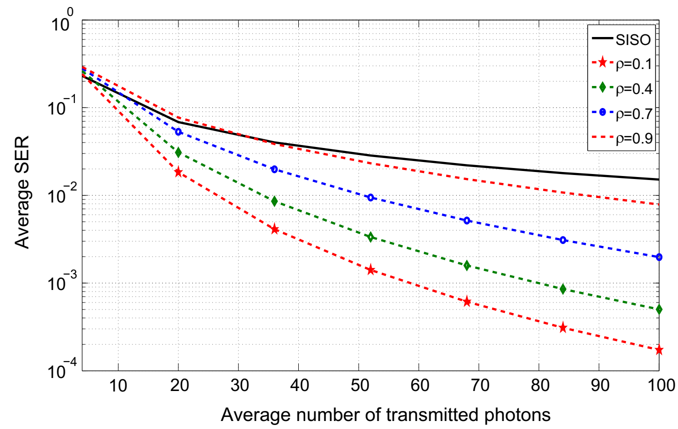

Fig. 4. Average SER versus average number of transmitted photons for an MPPM system with $Q=4$ and $w=2$ under strong turbulent channel with $\chi_{SC}^2 = 1.25$, $M = N = 2$, and different values of correlation coefficient ρ.

the average SER versus the average number of transmitted photons of a MIMO-FSO system adopting MPPM scheme for different values of correlation coefficients. As shown in figure, the higher the correlation coefficient the worse system performance. In the special case of high correlation coefficient, i.e., $\rho \approx 1$, all the channels are affected by same fading and the received signals are practically the same, which leads to a single effective channel. In this situation, both MIMO and SISO systems have nearly the same performance. In contrast, for lower correlation values, $\rho \approx 0$, all channels are nearly independent, taking complete advantage of the MIMO diversity concept and leading to a noticeable performance improvement.

6. Conclusion

The performance, in terms of symbol-error-rates, of shot-noise-limited MIMO-FSO systems adopting MPPM techniques has been studied. The atmospheric turbulence effects have been modeled by the widely used gamma-gamma distribution. The summation of the *MN* gamma-gamma random variables have been modeled by two robust pdf expressions for the two cases of uncorrelated and correlated channels. Exact SER expressions have been derived for both channel configurations. Our expressions have been used to investigate the MIMO system performance. In addition the obtained results have been compared to that of the equivalent SISO systems. Finally, the effect of the correlation coefficients on the system performance has also been studied, concluding as expected that to take advantage of the diversity technique, it is desirable for the received signals to be as independent as possible.

References

[1] S.S. Muhammad, T. Javornik, I. Jelovcan, E. Leitgeb, O. Koudelka, Reed solomon coded PPM for terrestrial FSO links, in: Proc. Int. Conf. on Elect. Engin. (ICEE 2007), Portugal, 2007, pp. 1–5.
[2] S. Bloom, E. Korevaar, J. Schuster, H. Willebrand, Understanding the performance of free-space optics, J. Opt. Netw. 2 (6) (2003) 178–200.
[3] X. Zhu, J.M. Kahn, Free-space optical communication through atmospheric turbulence channels, IEEE Trans. Commun. 50 (8) (2002) 1293–1300.
[4] K. Kiasaleh, Performance of APD-based, PPM free-space optical communication systems in atmospheric turbulence, IEEE Trans. Commun. 53 (9) (2005) 1455–1461.
[5] R.L. Al-Habash, M.A. Andrews, L.C. Phillips, Mathematical model for the irradiance probability density function of a laser beam propagating through turbulent media, Opt. Eng. 40 (8) (2001) 1554–1562.
[6] S.G. Wilson, M. Brandt-Pearce, Q. Cao, M. Baedke, Optical repetition MIMO transmission with multipulse PPM, IEEE J. Sel. Areas Commun 23 no. 9 (2005) 1901–1909.

[7] H. Sugiyama, K. Nosu, MPPM: a method for improving the band utilization efficiency in optical PPM, J. Lightw. Technol. 7 (3) (1989) 465–472.
[8] S.M. Navidpour, M. Uysal, M. Kavehrad, BER performance of free-space optical transmission with spatial diversity, IEEE trans. wireless commun. 6 (8) (2007) 2813–2819.
[9] H. Kazemi, M. Uysal, Performance analysis of mimo free-space optical communication systems with selection combining, in: Signal Processing and Communications Applications Conference (SIU), 2013 21st, 2013, pp. 1–4.
[10] N. Cvijetic, S.G. Wilson, M.B. Pearce, Performance bounds for free-space optical MIMO systems with APD receivers in atmospheric turbulence, IEEE J. sel. areas commun. 26 (3) (2008) 3–12.
[11] S. Jiang, G. Yang, Y. Wei, M. Bi, Y. Lu, X. Zhou, M. Hu, Q. Li, Performance analysis of space-diversity free-space optical links over exponentiated weibull channels, Photonics Technology Letters, IEEE PP (99).
[12] P. Kaur, V. Jain, S. Kar, Performance analysis of free space optical links using multi-input multi-output and aperture averaging in presence of turbulence and various weather conditions, Communications, IET 9 (8) (2015) 1104–1109.
[13] C. Abou-Rjeily, Performance analysis of fso communications with diversity methods: Add more relays or more apertures?, Selected Areas in Communications, IEEE Journal on PP (99) (2015) 1–1.
[14] H. Kazemi, Z. Mostaani, M. Uysal, Z. Ghassemlooy, Outage performance of MIMO free-space optical systems in gamma-gamma fading channels, in: NOC/OC&I, 2013.
[15] K.P. Peppas, A simple, accurate approximation to the sum of gamma-gamma variates and applications in MIMO free-space optical systems, IEEE Photon. Technol. Lett. 23 (July (13)) (2011) 839–841.
[16] J.M.G. Balsells, A.J. Navas, J.F. Paris, M.C. V´azquez, A.P. Notario, Spatially correlated gamma-gamma scintillation in atmospheric optical channels, Opt. Exp. 22 (18) (2014) 21820–21833.
[17] Z. Chen, S. Yu, T. Wang, G. Wu, S. Wang, W. Gu, Channel correlation in aperture receiver diversity systems for free-space optical communication, Journal of Optics 14 (2012) 125710–125716.
[18] M.D. Yacoub, The α-μ distribution: a physical fading model for the stacy distribution, IEEE Trans. Veh. Technol 56 (1) (2007) 27–34.
[19] A.E. Morra, H.S. Khallaf, H.M.H. Shalaby, Z. Kawasaki, Performance analysis of both shot- and thermal-noise limited multipulse PPM receivers in gamma-gamma atmospheric channels, J. Lightw. Technol 31 (19) (2013) 3142–3150.
[20] Wolfram Function Site. (2013, Feb.). [Online]. Available: 〈 http://functions.wolfram.com/〉.
[21] I.M. Gradshteyn, I.S. Ryzhik, Table of Integrals, Series and Products, seventh Edition.., Academic Press, 2007.
[22] M. Abramowitz, I.A. Stegun (Eds.), Handbook of mathematical functions, 10th Edition.,Dover Publications, 1972.